Model-Free Damage Identification of Skeletal Structures

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ABSTRACT

This paper presents and verifies experimentally a model-free methodology for off-line damage identification of truss structures. The Virtual Distortion Method (VDM) is used, which allows the approach to be based entirely on experimentally obtained non-parametric characteristics of the monitored structure, so that no parametric numerical modeling is necessary. The damage is modeled using certain damage-equivalent pseudo-loads, which are convolved with experimentally obtained local responses of the original structure to compute the response of the damaged structure. An effective sensitivity analysis is possible via the adjoint variable method.

INTRODUCTION

The long-term motivation for this research is the need for a practical technique for damage monitoring that could be used in black-box type monitoring systems. In general, most of the global SHM methods can be classified into three general groups [1–3]:

1. *Model-based methods*, which rely on a parametric numerical model of the monitored structure that is usually a Finite Element (FE) model [4–6] or a continuum model [7]. The identification is stated in the form of a minimization problem of the discrepancy between the measured response of the damaged structure and the computed response of the modeled structure.
2. *Pattern recognition methods* rely on a database of numerical fingerprints of low dimension that are extracted from several responses of the involved structure [8,9]. The responses used to form the database have to be previously collected either by simulations or by experimental measurements of the structure with introduced modification scenarios, which are to be identified later and which should be well discriminated by the fingerprints. Given the database and the measured response of the involved structure, the

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actual modification is identified using the fingerprints only, without insight into their actual mechanical meaning, so that neither a numerical model of the structure nor a simulation is required at the identification stage.

3. In case of many real-world structures, it may not be possible to actually introduce the modifications (in order to build the fingerprint database) or to build an accurate numerical model [10]. Therefore, there is a *third group of methods*, which rely on certain structural invariants that can be computed directly from the measured response and which can be modal [3], based on wavelet or time series analysis [11], use the response surface methodology [12] or Lyapunov exponents [13]. By a proper distribution of sensors in the structure, the invariants can be compared locally, which may allow the detected modification to be also localized.

In a sense, the approach proposed in this paper belongs to the third group, because it avoids actual modifications as well as parametric numerical modeling; however, it is aimed also at quantification of the modification as the methods of the first two groups. Identification of the modifications is formulated as an optimization problem of minimizing the discrepancy between the measured and the modeled structural responses. The VDM [14,15] is used, which allows the structure to be modeled in an essentially non-parametric way via its locally measured responses, which are limited to the potential modification points. Given the excitation and the measured response of the original undamaged structure, the corresponding response of the damaged structure is computed by using certain damage-equivalent pseudo-loads, which are convolved with the experimentally obtained local responses of the unaffected structure. The pseudo-loads are imposed on the undamaged structure to model the mass- and stiffness-related modifications; they are given in the form of the unique solution to a certain linear integral equation. The formulation makes possible an effective first- and second-order sensitivity analysis via the adjoint variable method.

The methodology is validated numerically and experimentally using a 4-meter-long, 70-element truss structure.

RESPONSE OF THE DAMAGED STRUCTURE

Let the original undamaged structure obey the following equation of motion:

$$\mathbf{M}\ddot{\mathbf{u}}^{\mathrm{L}}(t) + \mathbf{C}\dot{\mathbf{u}}^{\mathrm{L}}(t) + \mathbf{K}\mathbf{u}^{\mathrm{L}}(t) = \mathbf{f}(t), \tag{1}$$

where **f**(t) is a given excitation. Let the damage of the structure be described by modifications $\Delta\mathbf{M}$ and $\Delta\mathbf{K}$ to its mass and stiffness matrices and use **u**(t) to denote the response of the damaged structure to the same excitation **f**(t),

$$[\mathbf{M} + \Delta\mathbf{M}]\,\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + [\mathbf{K} + \Delta\mathbf{K}]\,\mathbf{u}(t) = \mathbf{f}(t). \tag{2}$$

The VDM [14,15] can be used to model modifications of structural mass and stiffness with a response-coupled field of certain pseudo-loads $\mathbf{p}^0(t)$, which act in the unmodified structure to imitate the effects of the modifications:

$$\mathbf{M}\ddot{\mathbf{u}}(t)+\mathbf{C}\dot{\mathbf{u}}(t)+\mathbf{K}\mathbf{u}(t)=\mathbf{f}(t)+\mathbf{p}^0(t), \qquad \text{where} \qquad \mathbf{p}^0(t)=-\Delta\mathbf{M}\ddot{\mathbf{u}}(t)-\Delta\mathbf{K}\mathbf{u}(t). \tag{3}$$

Therefore, the response of the modified structure to the load **f**(t) can be expressed in terms of the convolution with the system impulse-responses:

$$\begin{aligned} \mathbf{u}(t)&=\mathbf{u}^{\mathrm{L}}(t)+\textstyle\int_0^t \mathbf{B}^0(t-\tau)\mathbf{p}^0(\tau)\,d\tau \qquad \text{or} \qquad \mathbf{u}=\mathbf{u}^{\mathrm{L}}+\mathcal{B}^0\mathbf{p}^0,\\ \ddot{\mathbf{u}}(t)&=\ddot{\mathbf{u}}^{\mathrm{L}}(t)+\textstyle\int_0^t \ddot{\mathbf{B}}^0(t-\tau)\mathbf{p}^0(\tau)\,d\tau \qquad \text{or} \qquad \ddot{\mathbf{u}}=\ddot{\mathbf{u}}^{\mathrm{L}}+\ddot{\mathcal{B}}^0\mathbf{p}^0. \end{aligned} \tag{4}$$

The matrices $\mathbf{B}^0(t)$ and $\ddot{\mathbf{B}}^0(t)$ contain the structural impulse-response functions and the corresponding integral operators are denoted by $\mathcal{B}^0$ and $\ddot{\mathcal{B}}^0$. A substitution of (4) into the second equation of (3) yields the following system of Volterra linear integral equations with the unknown vector of the pseudo-loads $\mathbf{p}^0$:

$$\mathbf{p}^0+\left[\Delta\mathbf{K}\mathcal{B}^0+\Delta\mathbf{M}\ddot{\mathcal{B}}^0\right]\mathbf{p}^0=-\Delta\mathbf{K}\mathbf{u}^{\mathrm{L}}-\Delta\mathbf{M}\ddot{\mathbf{u}}^{\mathrm{L}}. \tag{5}$$

Such a system, due to the distributional terms in $\ddot{\mathbf{B}}^0(t)$, can be proved to be of the second kind and thus uniquely solvable, if the matrix $\mathbf{M}+\Delta\mathbf{M}$ is non-singular. Given the modifications $\Delta\mathbf{M}$ and $\Delta\mathbf{K}$, and the solution $\mathbf{p}^0$ to (5), the response of the damaged structure can be computed by (4). Notice that the impulse-responses need to be measured only locally, that is in the degrees of freedom (DOFs) related to the potential modifications, as in other DOFs the pseudo-loads $\mathbf{p}^0$ vanish.

However, in reality the exact impulse-responses are hardly available: one can measure only responses to excitations that last several steps. Nevertheless, these responses can be also used, provided the pseudo-loads can be expressed in the form of the following convolution:

$$p_i^0(t)=(q_i\star p_i)(t)=\textstyle\int_0^t q_i(t-\tau)p_i(\tau)\,d\tau, \tag{6}$$

where $q_i(t)$ is the actually applied non-impulsive excitation in the ith DOF that has to satisfy $q_i(t)=0$ for $t<0$ and $p_i(t)$ is a certain unknown function. Equation (6) can be collected for all involved DOFs and stated in the operator notation to take the following form:

$$\mathbf{p}^0=\mathcal{Q}\mathbf{p}, \tag{7a}$$

where $\mathcal{Q}$ is the corresponding diagonal matrix convolution operator. A substitution of (7) into (4) and (5) yields

$$\begin{aligned} \mathbf{u}&=\mathbf{u}^{\mathrm{L}}+\mathcal{B}\mathbf{p},\\ \ddot{\mathbf{u}}&=\ddot{\mathbf{u}}^{\mathrm{L}}+\ddot{\mathcal{B}}\mathbf{p}, \end{aligned} \tag{8}$$

$$\left[\mathcal{Q}+\Delta\mathbf{K}\mathcal{B}+\Delta\mathbf{M}\ddot{\mathcal{B}}\right]\mathbf{p}=-\Delta\mathbf{K}\mathbf{u}^{\mathrm{L}}-\Delta\mathbf{M}\ddot{\mathbf{u}}^{\mathrm{L}}, \tag{9}$$

where $\mathcal{B}=\mathcal{B}^0\mathcal{Q}$ and $\ddot{\mathcal{B}}=\ddot{\mathcal{B}}^0\mathcal{Q}$, so that, contrary to (4) and (5), all data necessary to form (8) and (9) can be directly measured.

DAMAGE IDENTIFICATION

The inverse problem is stated here in the standard form of a problem of minimization of the following objective function:

$$F(\Delta\mathbf{M},\Delta\mathbf{K}) = \tfrac{1}{2}\int_0^T \|\mathbf{d}(t)\|^2\,dt \qquad \text{where} \qquad \mathbf{d}(t) = \mathbf{u}^{\mathrm{M}}(t) - \mathbf{u}(t), \tag{10}$$

which is the mean-square distance between the measured and simulated responses of the damaged structure. It is minimized with respect to a chosen set of parameters that define the damage via $\Delta\mathbf{M}$ and $\Delta\mathbf{K}$. The method adjoint can be used for fast sensitivity analysis [16,17]. The derivative with respect to the αth parameter is

$$F_\alpha(\Delta\mathbf{M},\Delta\mathbf{K}) = \int_0^T \boldsymbol{\lambda}^{\mathrm{T}}(t)\,[\Delta\mathbf{K}_\alpha\mathbf{u}(t) + \Delta\mathbf{M}_\alpha\ddot{\mathbf{u}}(t)]\,dt, \tag{11}$$

where $\boldsymbol{\lambda}(t)$ is the vector of the adjoint variables, defined as the solution to

$$\left[\mathcal{Q}^{\mathrm{T}} + \mathcal{B}^{\mathrm{T}}\Delta\mathbf{K} + \ddot{\mathcal{B}}^{\mathrm{T}}\Delta\mathbf{M}\right]\boldsymbol{\lambda} = \mathcal{B}^{\mathrm{T}}\mathbf{d}. \tag{12}$$

If required, the second derivative can be computed as

$$\begin{aligned} F_{\alpha\beta}(\Delta\mathbf{M},\Delta\mathbf{K}) = \int_0^T \mathbf{u}_\alpha^{\mathrm{T}}(t)\mathbf{u}_\beta(t) + \boldsymbol{\lambda}^{\mathrm{T}}(t)\,\big[&\Delta\mathbf{K}_\alpha\mathbf{u}_\beta(t) + \Delta\mathbf{M}_\alpha\ddot{\mathbf{u}}_\beta(t) \\ &+ \Delta\mathbf{K}_\beta\mathbf{u}_\alpha(t) + \Delta\mathbf{M}_\beta\ddot{\mathbf{u}}_\alpha(t) + \Delta\mathbf{K}_{\alpha\beta}\mathbf{u}(t) + \Delta\mathbf{M}_{\alpha\beta}\ddot{\mathbf{u}}(t)\big]\,dt, \end{aligned} \tag{13}$$

where the derivatives of the responses are computed via differentiated (8) and (9),

$$\left[\mathcal{Q} + \Delta\mathbf{K}\mathcal{B} + \Delta\mathbf{M}\ddot{\mathcal{B}}\right]\mathbf{p}_\alpha = -\Delta\mathbf{K}_\alpha\mathbf{u} - \Delta\mathbf{M}_\alpha\ddot{\mathbf{u}}, \tag{14}$$

$$\mathbf{u}_\alpha = \mathcal{B}\mathbf{p}_\alpha, \qquad \ddot{\mathbf{u}}_\alpha = \ddot{\mathcal{B}}\mathbf{p}_\alpha. \tag{15}$$

Given the derivatives of the response, the first derivative of the objective function can be computed for verification purposes also by

$$F_\alpha(\Delta\mathbf{M},\Delta\mathbf{K}) = -\int_0^T \mathbf{d}^{\mathrm{T}}(t)\mathbf{u}_i(t)\,dt. \tag{16}$$

EXPERIMENT

A 32 kg 4-meter long 3D truss structure with 26 nodes and 70 steel elements was used in the experimental verification (Figure 1). A damage of one element (0.375 kg, *EA*=13 850 kN) was modeled by replacing it with an aluminum element of a comparable weight (0.300 kg), but reduced stiffness (*EA*=9 270 kN). A modal hammer was used to generate the influence matrix and the test loading **f**(t). The loading acted in the z-direction (perpendicularly to the plane marked blue in Figure 1), and the displacement response was measured in the same direction.

Figure 2 plots the objective function in dependence on the absolute stiffness reduction (the mass was assumed to remain the same). The minimum was found at 4290 kN, which is relatively close to the actual value of 4580 kN. The measured and computed responses are compared in Figure 3.

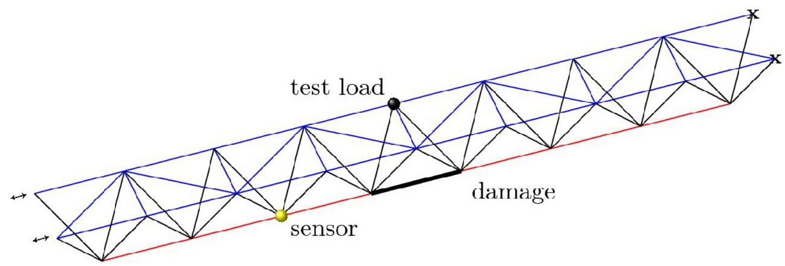

Figure 1. Truss structure used in the experimental verification

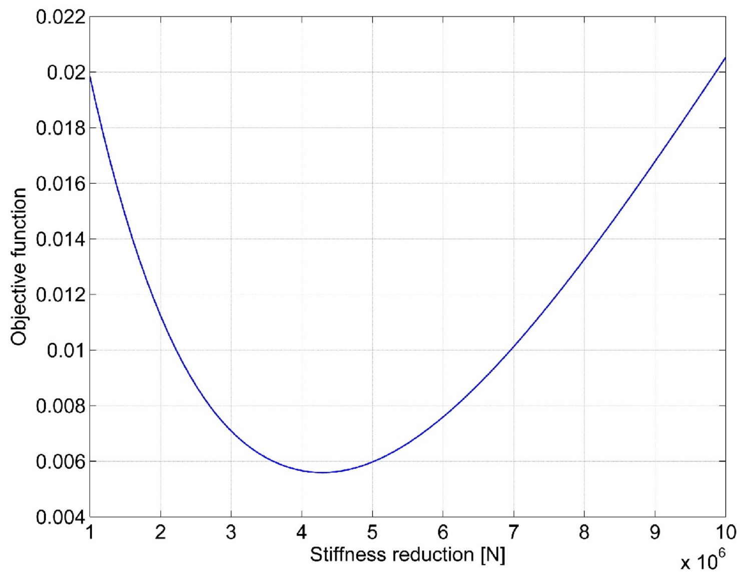

Figure 2. Objective function

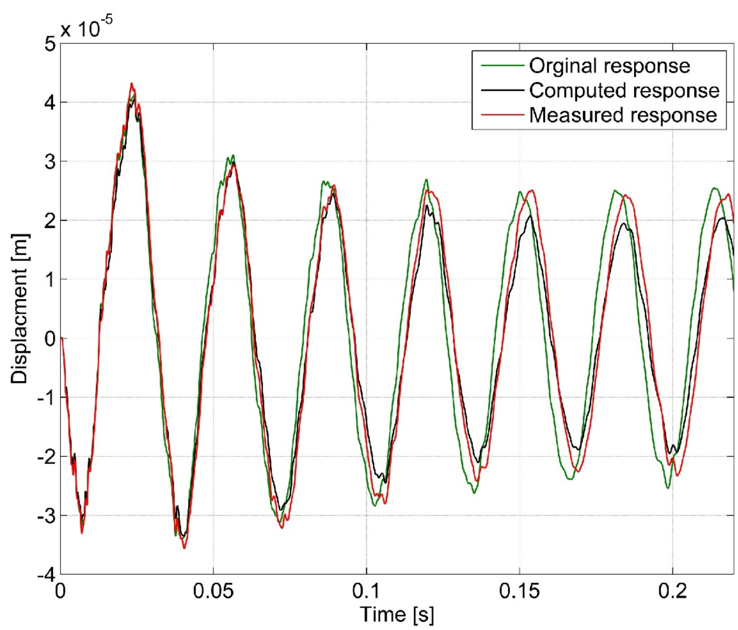

Figure 3. Measured responses of the original and damaged structures and the computed response

CONCUSIONS

This paper proposes and verifies experimentally a model-free approach to identification of damages in skeletal structures. The approach is based on the virtual distortion method (VDM), which allows experimentally measured local impulse-responses to be directly used to model the response of the damaged structure. As a result, no parametric numerical model of the structure is required.

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